

# On matrix-subadditive functions and a relevant trace inequality

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## Abstract

Let  $f$  be a real-valued function on  $[0, \infty)$  with  $f(0) = 0$  and  $n$  be a natural number greater than 1. We prove that if  $f$  is matrix-subadditive of order  $n$  then it has the form  $f(t) = \alpha t$  for some  $\alpha \in \mathbb{R}$ . Moreover, we show that if the inequality  $\text{Tr}(f(A + B)) \leq \text{Tr}(f(A)) + \text{Tr}(f(B))$  holds true for every pair  $A, B$  of Hermitian positive semidefinite  $n \times n$ -matrices then  $f$  is concave. © 1998 OPA (Overseas Publishers Association) Amsterdam B.V. Published under license under the Gordon and Breach Science Publishers imprint.

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## Keywords

Matrix-subadditive function, Trace inequality of subadditivity